

# A Simplified Time-Delayed Disturbance Observer for Position Control of Robot Manipulators\*

S. Jung, Y. G. Bae, and M. Tomizuka

**Abstract**—Disturbance observers (DOB) are used to reject external disturbances as well as inherent internal uncertainties. The time-delayed control method is also a kind of DOB that cancels out uncertainties by delayed information. Relying on a delayed control input may cause the phase delay in the system, and it further leads to the instability of the system. Another problem is to use an acceleration feedback which is so noisy that the performance may be degraded. In this paper, a simplified time-delayed DOB (STD DOB) is introduced to compensate for the weakness of using the acceleration information. To confirm the feasibility of STD DOB, simulation and experimental studies of position control of the two link robot manipulator is performed and evaluated.

## I. INTRODUCTION

The time-delayed control method has been popular in 1980's, then it seems that research interest on the time-delayed control (TDC) is gradually downsizing. Although it has a simple structure to use delayed information to cancel out uncertainties, popularity is getting mistier [1,2].

Roughly saying, there are two reasons why the time-delayed control method loses researchers' attraction in the control community. One is the stability of the closed loop system, which considers the most important fact that should be satisfied first. Taking the delayed outputs and feeding them back to the control loop often raise the instability issue. Another drawback of the TDC is the use of acceleration measurements to estimate dynamics and other uncertainties by the virtue of the algorithm.

It is true that direct measurement of acceleration is not practical due to the sensor cost or no availability. They often use a simple calculation from position measurements to estimate acceleration, which forms the second order filter.

In the meanwhile, DOB proposed by Ohnishi has attained attention from industrial-oriented motion control systems [3]. DOB has a simple structure to reject disturbance effectively without any cost. A design method of two-degrees-of-freedom controller is presented and applied to the digital motion control system [4-7]. In industries, DOB is often used for high speed motion control systems such as disk drive control systems [8-11].

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In the framework of the DOB structure, the difference comes from the way of estimating disturbance. DOB requires the inverse model of the plant to extract disturbance from the output. The output passes through the inverse model of the plant to identify the control input including disturbance, and then compared with the nominal control input to exclude the disturbance, and finally subtracted it from the control input again to cancel out the real disturbance by the estimated one.

Therefore, DOB is an inverse model-based control scheme, which carries many difficulties as well. Firstly, poor modeling has a problem. Modeling errors may result in a poor estimation of disturbance and lead to poor disturbance rejection, even instability. Secondly, the inverse model of the plant may not exist if the plant is non-minimum phase [12]. Thirdly, since DOB is based on the inverse model of the plant, it is not appropriate for the non-linear systems, whose modeling is quite difficult.

In a meanwhile, since a TDC method is a partial model-based approach and can be easily used for non-linear systems, it is still worthwhile to investigate further. It is promising that a sampling time for the control system is getting faster as the technology develops further.

Experimental studies for the position control of robot manipulators using the time-delayed control method have been presented [13]. Recently, acceleration-based control in DOB structure has been applied to motion control systems [14].

For robot manipulators, it is not easy to apply the current DOB since the system is nonlinear and multi-input multi-output. In this paper, starting from the ideal time-delayed DOB structure for a linear system, the simplified time-delayed DOB (STD DOB) structure is presented. In the STD DOB structure, feedback from acceleration measurement is ignored. If the robot moves at a constant speed, then acceleration becomes zero, which is true for many robots. The ignorance of an acceleration term can be compensated by simply increasing the constant controller gain in the feedback loop. The proposed STD DOB is applied to control position of a two-link robot manipulator to prove the feasibility.

## II. ROBOT DYNAMICS

The dynamic equation of an  $n$  degrees-of-freedom robot manipulator in joint space coordinates is given by :

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where the vectors  $q$ ,  $\dot{q}$ ,  $\ddot{q}$  are the  $n \times 1$  joint angle, the  $n \times 1$  joint angular velocity, and the  $n \times 1$  joint angular acceleration, respectively,  $M(q)$  is the  $n \times n$  symmetric positive definite inertia matrix,  $C(q, \dot{q})$  is the  $n \times 1$  vector of Coriolis and centrifugal torques,  $G(q)$  is the  $n \times 1$  gravitational torques, and  $\tau$  is the  $n \times 1$  vector of actuator joint torques.

For simplicity, let us denote  $H = C(q, \dot{q})\dot{q} + G(q)$  so that equation (1) can be rewritten as

$$M(q)\ddot{q} + H = \tau \quad (2)$$

Define the tracking error as

$$e = q_d - q \quad (3)$$

where  $q_d$  is the desired trajectory and  $q$  is the actual trajectory.

Selecting PD control input yields the control law as

$$\tau = \hat{M}(q)\bar{U}(t) \quad (4)$$

where  $\hat{M}$  is the estimation of  $M$  and the control input  $\bar{U}(t)$  is given by a PD controller

$$\bar{U}(t) = K_D \dot{e} + K_P e \quad (5)$$

where  $K_D, K_P$  are controller gain matrices.

Combining (2), (4) and (5) yields the closed loop error equation.

$$\ddot{e} + M^{-1}\hat{M}(K_D \dot{e} + K_P e) = M^{-1}(M\ddot{q}_d + H) \quad (6)$$

We see from (6) that the tracking error is affected by the inertia matrix  $M$ , its estimate,  $\hat{M}$ , and other unknown dynamic terms.

### III. DESIGN OF TIME-DELAYED DOB

Consider a single-input single-output system of (2). Here we have the second order dynamic system.

$$m\ddot{y}(t) + h(t) = u(t) \quad (7)$$

where  $m$  is a mass,  $y(t)$  is the output, and  $h(t)$  includes other dynamical terms including disturbance, and  $u(t)$  is the control input. A simple way to estimate  $h(t)$  is to use the previous information of the system dynamics, which is known as a time-delayed control method.

$$\hat{h}(t) = u(t - T) - \hat{m}\ddot{y}(t - T) \quad (8)$$

where  $T$  is a sampling time and  $\hat{m}$  is an estimated mass. Therefore, the desired control law becomes

$$\begin{aligned} u(t) &= \hat{m}\bar{u}(t) + \hat{h}(t) \\ &= \hat{m}\bar{u}(t) + u(t - T) - \hat{m}\ddot{y}(t - T) \end{aligned} \quad (9)$$

where

$$\bar{u}(t) = k_d \dot{e} + k_p e \quad (10)$$

Note that the time-delayed control is named after using previously sampled information for the current control input as in (9).

Performing Laplace transform of (9), the control input can be described as

$$U(s) = \frac{\hat{m}}{1 - e^{-sT}} (\bar{U}(s) - s^2 e^{-sT} Y(s)) \quad (11)$$

The complete control block diagram is shown in Fig. 1.

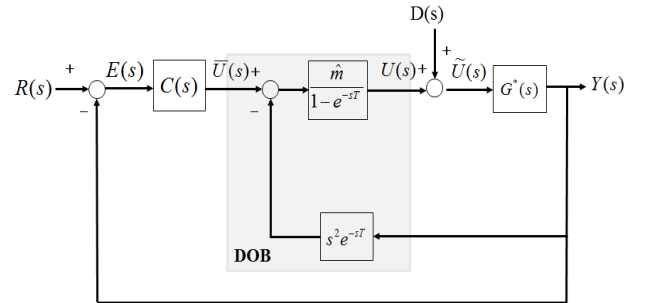


Fig.1 Time-delayed DOB with indirect acceleration feedback

The closed loop transfer function of Fig.1 is obtained as

$$Y(s) = \frac{\hat{m}G^*(s)C(s)R(s)}{1 - e^{-sT} + \hat{m}s^2 e^{-sT} G^*(s) + \hat{m}G^*(s)C(s)} + \frac{G^*(s)(1 - e^{-sT})D(s)}{1 - e^{-sT} + \hat{m}s^2 e^{-sT} G^*(s) + \hat{m}G^*(s)C(s)} \quad (12)$$

where  $D(s)$  is the disturbance.

Note from (12) that the fast sampling time guarantees the stability and the disturbance effect is eliminated. It is known that feedback from the acceleration measurement is quite noisy when obtained in the structure of Fig. 1, and acceleration feedback is minimal in some applications when the robot is moving at a constant velocity.

The structure of Fig.1 can be modified as Fig. 2 based on the assumption that an acceleration measurement is directly available by using sensors. Fig. 2 shows a further simplified structure of Fig. 1 that is more suitable for control of a robot manipulator.

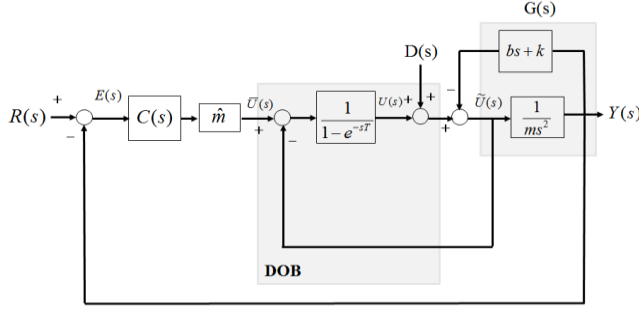


Fig.2 Time-delayed DOB with direct acceleration feedback

Therefore, our proposal is not to use the acceleration feedback term as shown in Fig. 3. Then the closed loop equation becomes

$$Y(s) = \frac{\hat{m}G^*(s)C(s)}{1-e^{-sT} + \hat{m}G^*(s)C(s)}R(s) + \frac{G^*(s)(1-e^{-sT})}{1-e^{-sT} + \hat{m}G^*(s)C(s)}D(s) \quad (13)$$

Note that (13) becomes dead-beat control and disturbance effect is eliminated when  $T=0$ .

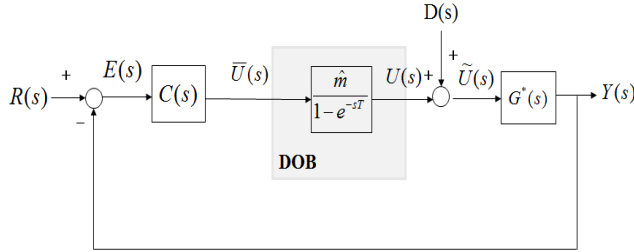


Fig.3 Simplified TD DOB like structure of Fig. 1 and 2

#### IV. TIME-DELAYED DOB FOR ROBOT CONTROL

##### A. Direct Acceleration Feedback

The original TD DOB structure in Fig. 1 requires the measurement of acceleration and feedback of it. The common way of obtaining the acceleration is to approximate from the joint measurement through the filter.

When sensors are used, however, direct measurement is possible as shown in Fig. 4. The measured acceleration values are fed back to form the control law

$$\tau(t) = \hat{M}\bar{U}(t) + \tau(t-T) - \hat{M}\dot{q}(t) \quad (14)$$

where

$$\bar{U}(t) = \ddot{q}_d(t) + K_D\dot{e}(t) + K_P e(t) \quad (15)$$

Combining (14) and (15) with the robot dynamics equation yields the closed error equation.

$$\ddot{e}(t) + K_D\dot{e}(t) + K_P e(t) = \hat{M}^{-1}(\tau(t) - \tau(t-T)) \quad (16)$$

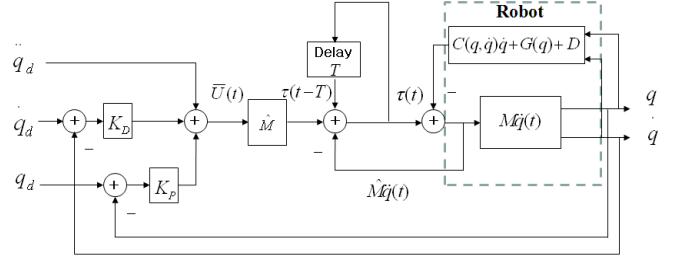


Fig.4 Scheme 1: TD DOB structure for robot control with direct acceleration feedback

##### B. Ignorance of Acceleration feedback

The concept of the simplified TD DOB can be applied to the robot control. Eliminating the acceleration term in (14) yields the simplified control law as

$$\tau(t) = K\hat{M}\bar{U}_s(t) + \tau(t-T) \quad (17)$$

where  $K$  and  $\hat{M}$  is a constant matrix which is assumed to be the best estimate of  $M$  and  $\bar{U}_s(t)$  is the feedback PD control.

$$\bar{U}_s(t) = K_D\dot{e}(t) + K_P e(t) \quad (18)$$

Combining (17) and (18) with the robot dynamics equation yields the closed loop error equation.

$$K_D\dot{e}(t) + K_P e(t) = \hat{M}^{-1}K^{-1}(\tau(t) - \tau(t-T)) \quad (19)$$

Since  $\hat{M}$  is a constant matrix, it also plays a role of increasing control gains such as  $K\hat{M}$ . A user can design  $K\hat{M}$  such that the system can satisfy the specification, which turns out to be high gain feedback control.

For the  $i$ th joint, we have the equation as

$$k_{di}\dot{e}_i(t) + k_{pi}e_i(t) = \frac{1}{\hat{m}_i k_i}(\tau_i(t) - \tau_i(t-T)) \quad (20)$$

Taking the Laplace transform of (20) yields

$$\frac{E_i(s)}{\tau_i(s)} = \frac{(1-e^{-sT})}{\hat{m}_i k_i (k_{di}s + k_{pi})} \quad (21)$$

which converges to zero.

The detailed control block diagram of the simplified TD DOB is shown in Fig. 5.

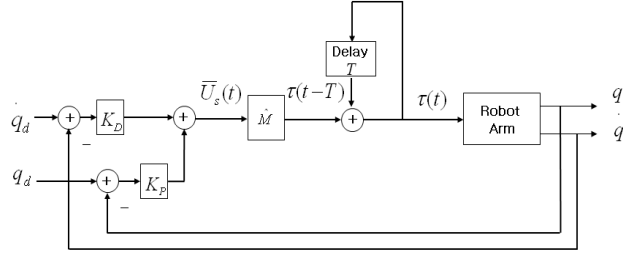


Fig.5 Scheme 2: Simplified TD DOB structure for robot control

## V. SIMULATION STUDIES

For simulation studies, a two-link robot manipulator is used. PD controllers are used to satisfy the tracking performance. There are two uncertainties given to the robot manipulator. First, friction at each joint is given. Friction terms at each joint are given as

$$f = 5\text{sgn}(\dot{q}) + 3\dot{q} \quad (22)$$

Fig. 6 shows the applied friction at each joint, which is considered as inherent uncertainties of the system. Second, the model is not exact such that  $M \neq \hat{M}$ . The inertia matrix is a function of joint variables, which is changed with respect to the joint configuration. For two degrees-of-freedom robot, the inertia matrix  $M$  can be described as

$$M = \begin{bmatrix} (m_1 + m_2)l_1^2 + 2m_2l_1l_2c_2 + m_2l_2^2 & m_2l_2^2 + m_2l_1l_2c_2 \\ m_2l_2^2 + m_2l_1l_2c_2 & m_2l_2^2 \end{bmatrix} \quad (23)$$

where  $m_1, m_2$  are link mass,  $l_1, l_2$  are link length, and  $c_2 = \cos(\theta_2)$ .

In the control law, the estimated inertia matrix,  $\hat{M}$  is assumed to be constant by excluding terms associated with joint variables.

$$\hat{M} = \begin{bmatrix} (m_1 + m_2)l_1^2 + m_2l_2^2 & m_2l_2^2 \\ m_2l_2^2 & m_2l_2^2 \end{bmatrix} = \begin{bmatrix} 2.4 & 0.8 \\ 0.8 & 0.8 \end{bmatrix} \quad (24)$$

where the robot parameters are given as  $m_1 = m_2 = 5\text{Kg}$  and  $l_1 = l_2 = 0.4\text{m}$ . Then the robot is required to move the goal position from the initial position,  $q = [0.7854 \quad -0.7854]^T$ . The desired goal position is  $q_d = [1.355 \quad 0.7854]^T$ . PD controller gains are selected as  $K_p = \text{diag}[80 \quad 80]^T$  and  $K_D = \text{diag}[40 \quad 40]^T$ .

### 1) Uncompensated case, PD control

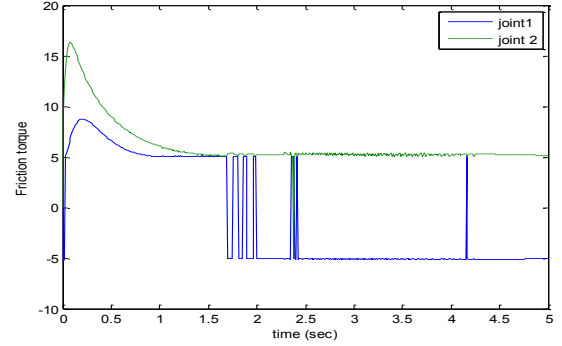
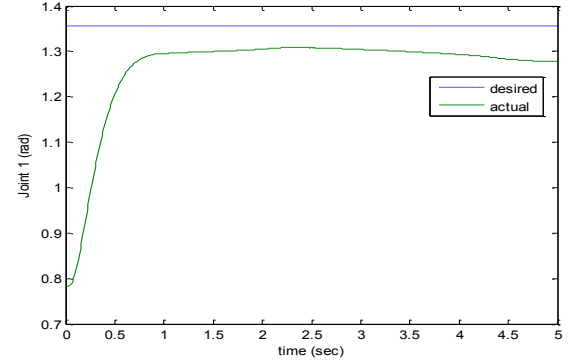
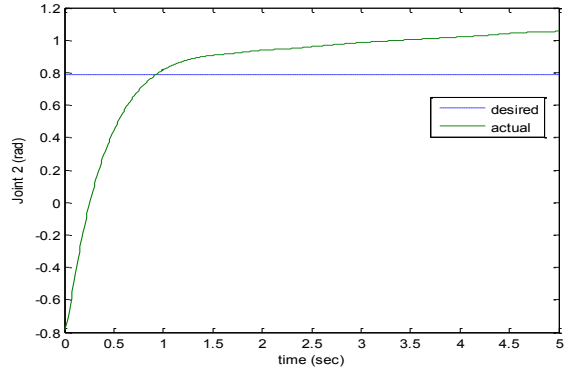


Fig. 6 Friction torque

Fig. 7 shows the position tracking response when the disturbance is not compensated. Position tracking errors are clearly present in both joints, which are affected by joint friction in (22) and a modeling error in (24) and other non-modeled terms.



(a) Joint 1



(b) Joint 2

Fig.7 Uncompensated Case

### 2) Scheme 1: TD DOB

When the control law of (14) in TD DOB is used, tracking errors due to modeling errors and friction are compensated as shown in Fig. 8. Note that here acceleration  $\ddot{q}(t)$  is assumed to

be available and the inertia is modeled as  $\hat{M}$ .

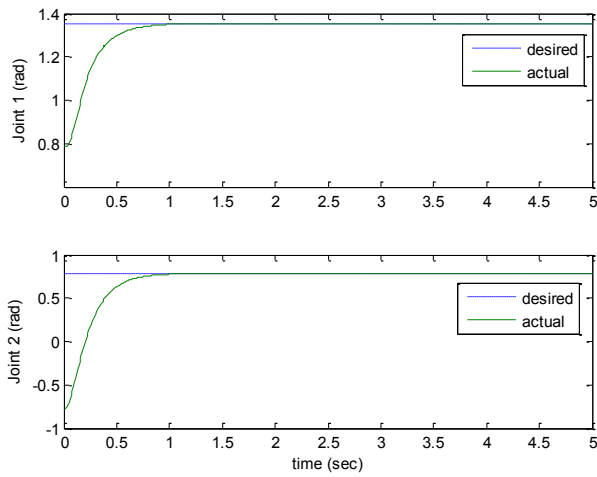


Fig.8 TD DOB with direct measurements of  $\hat{M}\dot{q}(t)$

3) *Scheme 2: STD DOB*

Next simulation is when STD DOB is applied. Exact inertia model in (23) is assumed ( $M = \hat{M}$ ), but other dynamic terms are still unknown. Position tracking results are shown in Fig. 9. We observe the oscillation in the transient period, but each joint follows the desired position after 2 seconds. This oscillation appears due to the lack of acceleration feedback.

The same simulation is conducted by using the constant inertial matrix given in (24). Fig. 10 shows the step response for both joints. Comparing with Fig. 9, we see the deviated error in both joints minimized. Since the acceleration feedback is not used, oscillations in the transient period are observed.

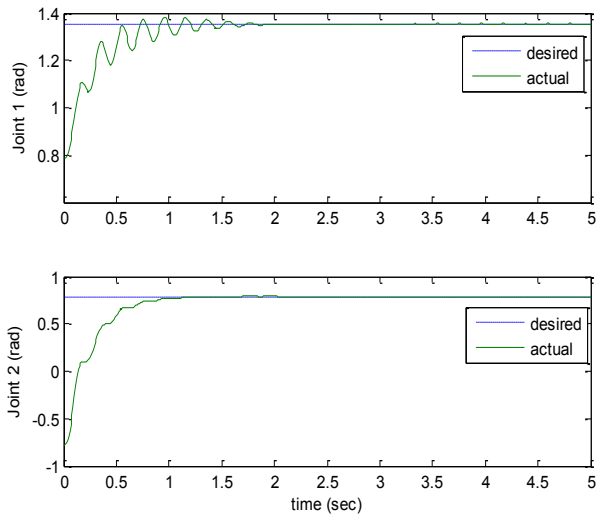
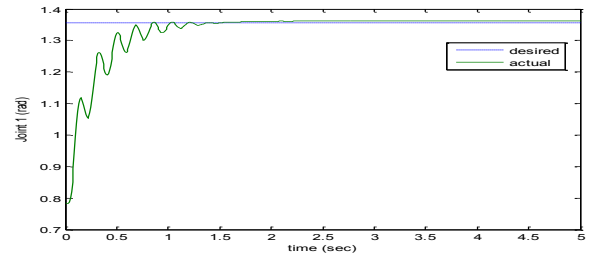
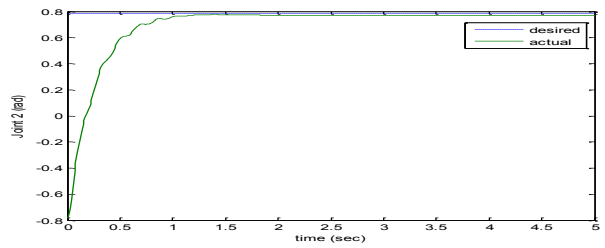


Fig.9 Compensated by STD DOB when an exact inertia model is used ( $M = \hat{M}$ )



(a) Joint 1



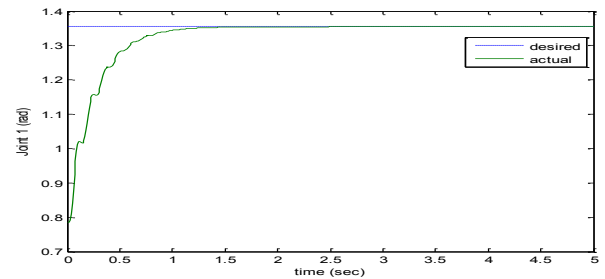
(b) Joint 2

Fig.10 Compensated by STD DOB : $k=1$

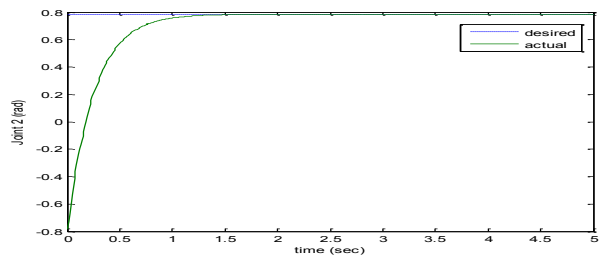
Then, the tracking errors can further be minimized by the gain  $k$ . The second stage control is to increase the gain  $k$  to compensate for disturbance.

4) *Scheme 3: STD DOB+K*

Fig. 11 shows the step responses when  $k$  is set to 2. The oscillatory behavior in the transient period is much reduced and steady state errors become zero after 1.5 seconds. If  $k$  is further increased to 3, then tracking performances are much better. This means that STD DOB can be used in practice without acceleration feedback.



(a) Joint 1



(b) Joint 2

Fig.11 Compensated by STD DOB : $k=2$

## VI. EXPERIMENTAL RESULTS

The proposed controller has been tested with two link robot arm. Joint 1 and joint 2 are commanded to move 45 degrees and -30 degrees, respectively. The gain value  $k_1 = 1, k_2 = 1$  for each joint of STD method is used, respectively. The estimation values of inertia are  $\hat{m}_1 = 0.3, \hat{m}_2 = 0.5$ . The corresponding results are plotted in Fig. 12. We see from the tracking error plot of Fig. 13 that tracking performances of time-delayed DOB and simplified TD DOB methods are comparable.

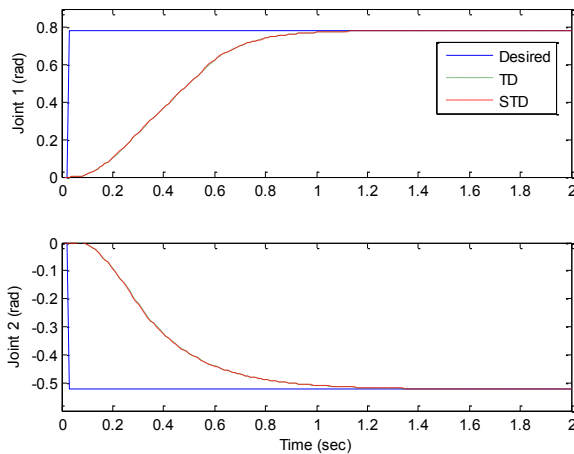


Fig.12 Tracking results by TD and STD methods

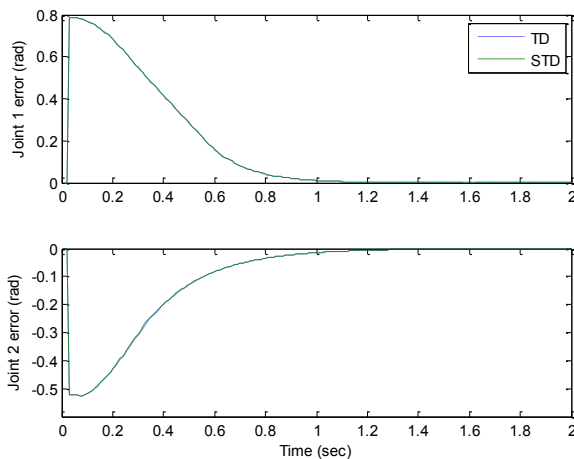


Fig.13 Tracking error results by TD and STD methods

## VII. CONCLUSION

The simple time-delayed DOB is applied to the position control of a robot manipulator. The STD DOB does not require any model except the constant inertia matrix. Although the acceleration feedback term is not used in the controller, uncertainties such as friction and unknown

dynamics are compensated by increasing the gain associated with the inertia model. It turns out that STD DOB functions as a high gain feedback control scheme, which means that it can also have the inherent problems of high gain feedback control. However, in the many industries, they still rely on the high gain feedback control under the stable condition.

## ACKNOWLEDGMENT

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