

# A Fuzzy Compensator Design for a Single-wheel Robot based on Static Instability

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**Abstract.** In this paper, the gyroscopic instability of a monocycle-like single-wheel robot is analyzed by means of the inverted stick model with vertical oscillation. To analyze the instability problem of the system, its dynamic model is described as a Mathieu's equation-like equation. A fuzzy logic control method is employed to compensate for the instability. The performance of the proposed compensator is verified by simulation studies.

**Keywords:** Fuzzy logic compensation, instability, a single-wheel robot.

## 1 Introduction

GYROBO, developed at Chungnam National University, is similar to a monocycle system that contains a gimbal system within a wheel. At a low speed or in a standing condition of GYROBO, the gimbal system is laterally required for a continuous balance control performance. However, when the state of the gimbal goes over the controllable boundary, the gyroscopic effect has a pitch-dominant property instead of yaw or roll-dominant property [1-3].

In this paper, the static stability problem of the robot can be studied using the inverted stick model with vertical oscillation property. A fuzzy logic compensation method is proposed. The physical properties of the vertical oscillation are experimentally analyzed.

After the fuzzy logic controller is designed using Simulink toolbox, the expected output is investigated. The fuzzy inference logic is built in the simulation of the balancing control of the single-wheel robot. Simulation of controlling the balance has been performed.

## 2 Inverted Stick Model of a Single-wheel System

In this model in Fig. 1, we use an inverted stick model with longitudinal oscillation property to analyze the instability properties of the robot system. The governing dynamic equation is

$$\frac{1}{6}ML\ddot{\theta} + (-MG + Mk\omega^2\cos\omega t)\sin\theta = F_L \quad (1)$$

$$\ddot{\theta} + \left(-\frac{6g}{L} + \frac{6}{L}k\omega^2\cos\omega t\right)\sin\theta = K_p e + K_d \dot{e} \quad (2)$$

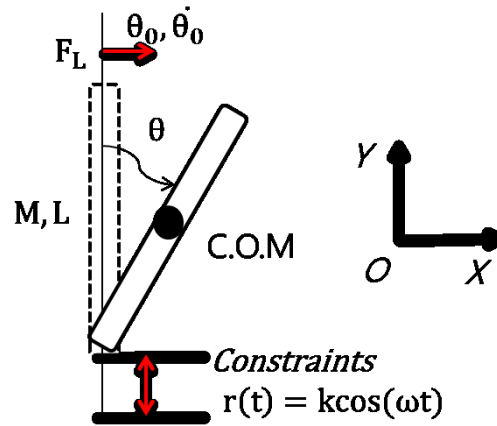


Fig. 1. Dynamic model

In case of  $K_p$  and  $K_d$  are zero in (2), the governing equation can be replaced with

$$\frac{d^2\theta}{d\tau^2} + \left(-\frac{\omega_0^2}{\omega^2} + \frac{6}{L}k\cos\tau\right)\sin\theta = 0 \quad (3)$$

Equation (3) shows Mathieu's equation-like equation [4-5].

### 3 Fuzzy Control Design

In the fuzzification process, a real scalar value of oscillation parameters can be translated into fuzzy sets. Table 1 shows the fuzzy input translation results of the proposed FLC. We assume that the gimbal states are linearly proportional to the oscillation parameter. NB is negative big, NM is negative medium, NS is negative small, ZO is zero, PS is positive small, PM is positive medium, and PB is positive big.

Table 1. Error fuzzification

Inputs	Crisp inputs (degrees)	Fuzzy variables
Error	-40 to -30	NB
	-30 to -20	NM
	-20 to -10	NS
	-10 to 10	ZO
	10 to 20	PS

	20 to 30	PM
	30 to 40	PB
Error rate	-2.5 to 2.5	ZO
	-5 to -2.5	NS
	-10 to -5	NM
	2.5 to 5	PS
	5 to 10	PM

Based on the PD control framework, the fuzzy compensator is added and the fuzzy output is added to the reference input which is the balancing angle.

$$u = K_p(\theta_d - \theta_f - \theta_{os}) + K_d(\dot{\theta}_d - \dot{\theta}_f) \quad (4)$$

where  $\theta_d$  is a desired angle,  $\dot{\theta}_d$  is a desired rate,  $\theta_{os}$  is FLC driven offset angle,  $K_p$  is a P-gain,  $K_d$  is a D-gain,  $\theta_f$  is a current angle,  $\dot{\theta}_f$  is a rate error,  $\theta_g$  is a gimbal angle,  $\dot{\theta}_g$  is a gimbal rate, and  $u$  is a torque input.

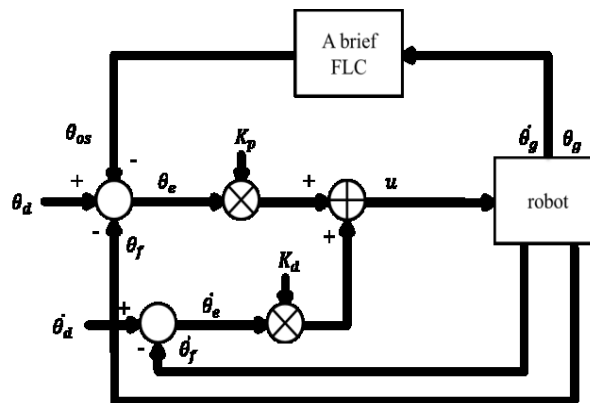
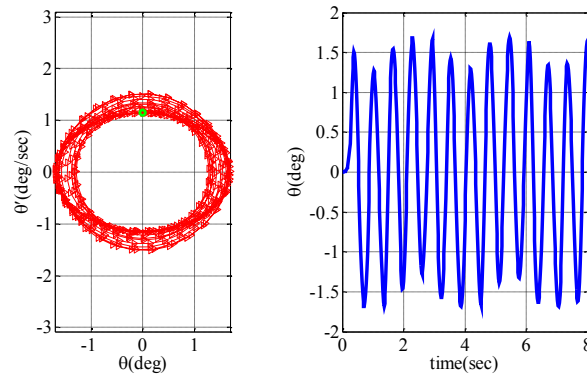


Fig. 2. Control block diagram

## 4 Simulation

The simulation is performed to verify if FLC is an adequate choice to the proposed problem of the instability. Simulation result of balancing control performance is shown in Fig 3. The instability can be remedied by the fuzzy offset compensation as shown in Fig. 3.



**Fig. 3. Balancing control performances**

## 5 Conclusions

In this paper, the static model of a single-wheel robot with vertical oscillation was presented and analyzed for the instability problem of the system. FLC was designed based on previous experiences. The determined offset values from the fuzzy logic controller were used for compensating for the instability. The simulation results were illustrated by the phase portrait and angle balancing. As a future work, experimental studies will be conducted to verify its performance.

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